Chapter 1

Kinematics

1.1 Motion Diagrams

When first applying kinematic (motion) principles, there is tendency to use the wrong kinematic quantity – to inappropriately interchange quantities such as position, velocity, and acceleration. Constructing a motion diagram should reduce this confusion and should provide a better intuitive understanding of the meaning of these quantities.

A motion diagram represents the position, velocity, and acceleration of an object at several different times. The times are usually separated by equal time intervals. At each position, the object's velocity and acceleration are represented by arrows. If the acceleration is constant throughout the motion, one arrow can represent the acceleration at all positions shown in the diagram.

You should be come so familiar with motion diagrams that you can read a linear-motion problem and draw a reasonable diagram that represents the motion described in the problem. When you complete the mathematical solution to a kinematic problem later in the semester, you can see if your answer is consistent with the motion diagram.

The Motion Diagrams for three common types of linear motion are described below.

1.1.1 Constant Velocity

The first motion diagram, shown in Figure 1.1, is for an object moving at a constant speed toward the right. The motion diagram might represent the changing position of a car moving at constant speed along a straight highway. Each dot indicates the position of the object at a different time. The times are separated by equal time intervals. Because the object moves at a constant speed, the displacements from one dot to the next are of equal length. The velocity of the object at each position is represented by an arrow with the symbol v under it. The velocity arrows are of equal length (the velocity is constant). *The acceleration is zero because the velocity does not change.*

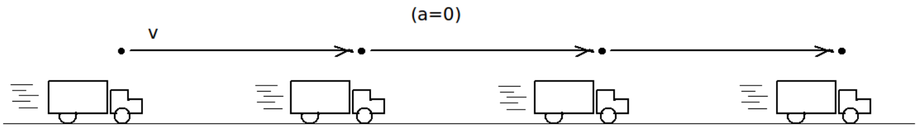


Figure 1.1: The motion diagram for an object moving with a constant velocity. The acceleration is zero because the velocity is not changing.

1.1.2 Constant Acceleration in the Direction of Motion

The motion diagram in Figure 1.2 represents an object that undergoes constant acceleration toward the right in the same direction as the initial velocity. This occurs when your car accelerates to pass another car or when a race car accelerates (speeds up) while traveling along the track. Once again, the dots represent schematically the positions of the object at times separated by equal time intervals Δt. Because the object accelerates toward the right, its velocity arrows increase in length toward the right as time passes. The product $a(\Delta t) = \Delta v$ represents the increase in length (increase in speed) of the velocity arrow in each time interval Δt. The displacement between adjacent positions increases as the object moves right because the object moves faster as it travels right.

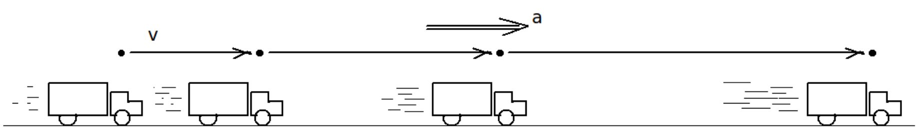


Figure 1.2: The motion diagram for an object accelerating in the direction of its velocity. The velocity increases as time progresses.

1.1.3 Constant Acceleration Opposite the Direction of Motion

The motion diagram in Figure 1.3 represents an object that undergoes constant accelera-

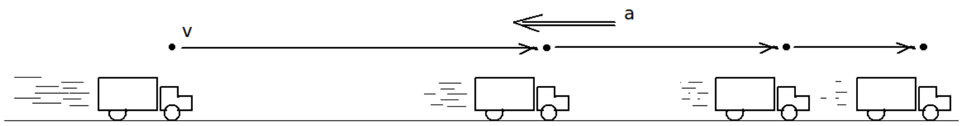
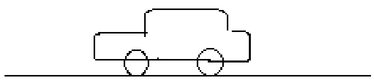
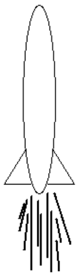
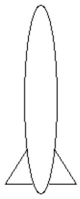
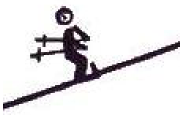
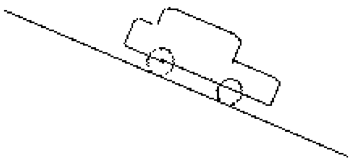
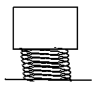


Figure 1.3: The motion diagram for an object accelerating in the opposite direction of its velocity. The velocity decreases as time progresses.

tion opposite the direction of the initial velocity (this is sometimes called "deceleration", a slowing of the motion). For this case, the acceleration arrow points left, opposite the direction of motion. This type of motion occurs when a car skids (打滑) to a stop. The dots represent schematically the positions of the object at equal time intervals. Because the acceleration points left, opposite the motion, the object's velocity arrows decrease by the same amount from one position to the next. We are now subtracting $\Delta v = a(\Delta t)$ from the velocity during each time interval Δt. Because the object moves slower as it travels right, the displacement between adjacent positions decreases as the object moves right.

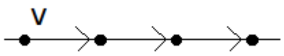
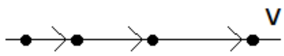
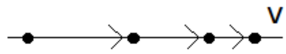
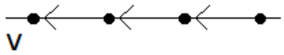
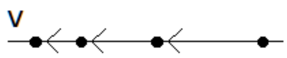
Table 1.1: Construct qualitative motion diagrams for each of the following situations:

(a) a car traveling toward the left at decreasing speed. 	(b) a rocket whose burning fuel causes it to move vertically upward at increasing speed. 	(c) the rocket in (b) after its fuel is burned up, and while it still moves upward, but now at decreasing speed. 
(d) a skier moving at decreasing speed up an incline. 	(e) a car coasting down an incline at increasing speed. 	(f) a block being pushed upward with increasing speed by a spring. 

More practice making motion diagrams:

- a bullet shot horizontally to the right, slowing down slightly because of wind resistance
- a turtle walking slowly but steadily northward
- a ball dropped from a tall building, moving down with a steady speed

Table 1.2: Given the velocity parts of the qualitative motion diagrams, first sketch and label the acceleration arrow, then construct a verbal description of the motion.

(a) 	(b) 	(c) 
(d) 	(e) 	(f) 

More practice: Consider each of the above, but tilted at some angle (as with an inclined plane). Then, imagine an object moving like that and write down phrases describing those situations.

1.2 Determining the Direction of Acceleration

The direction (and magnitude) of an object's acceleration at any time t can be determined if we know its velocity just before and just after that time.

By example, the procedure for graphically finding the direction is given here. Consider a general situation where both the speed and direction are changing.

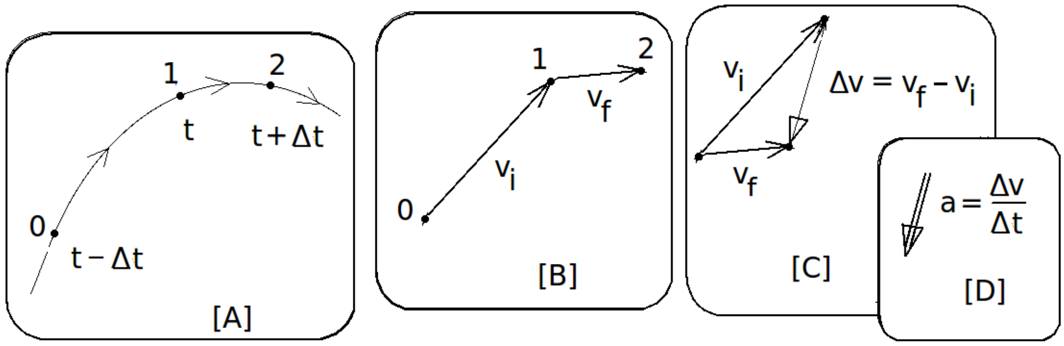


Figure 1.4: Subtracting velocities to determine the acceleration.

General Situation: As shown in [A], the object moves along the curved path, and slows down. Three locations are marked, numbered 0, 1, and 2. There is an equal time interval Δt between these points. We want to find the acceleration in the vicinity of point 1, at time t.

Initial Velocity v_i: To represent how it moves from dot 0 to 1, that is, before time t, draw the arrow and label it v_i.

Final Velocity v_f: To represent how it moves from dot 1 to 2, that is, after time t, draw the arrow and label it v_f.

Change in Velocity Δv: To represent the change in velocity, first redraw the arrows – putting the tails of v_f and v_i together. Then construct a third arrow pointing from the head of the initial to the head of the final. Call this third arrow Δv.

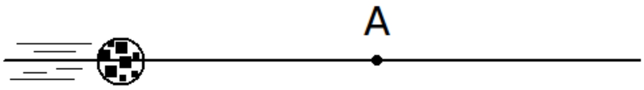
$$\overrightarrow{\Delta v} = \overrightarrow{v_f} - \overrightarrow{v_i} \qquad\qquad \overrightarrow{v_f} = \overrightarrow{v_i} + \overrightarrow{\Delta v}$$

Notice that Δv is the arrow that one must 'add' to v_i to get v_f.

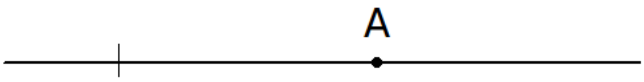
Acceleration a: The acceleration $\vec{a}$ is precisely $\overrightarrow{\Delta v} \div \Delta t$, but for now, we only want its direction: Always, both a and Δv point in the same direction. Draw and label the a arrow.

Graphically subtracting v's to find direction of a - 1

A ball moves with decreasing speed toward the right along a straight line. Determine the direction of the ball's acceleration when at position A.



Initial (earlier, before) Velocity: Draw an arrow representing the velocity of the ball before it arrives at point A. Label it v_i.



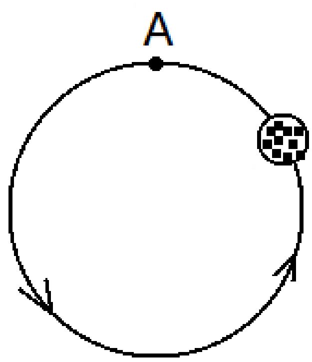
Final (later, after) Velocity: Draw an arrow representing the velocity of the ball after it passes point A. Label it v_f.

Velocity Change: Subtract the two velocity vectors to find $\overrightarrow{\Delta v} = \overrightarrow{v_f} - \overrightarrow{v_i}$. Put the tails together and draw the arrow that connects the head of v_i to the head of v_f. Label this new arrow Δv, "the change in velocity".

Acceleration: The acceleration equals the velocity change Δv divided by the time interval (between dots in the motion diagram). In this example, we only want the direction of a, which is <u>always</u> the same as the direction of Δv.
Go back to the original picture and draw an acceleration arrow and label it.

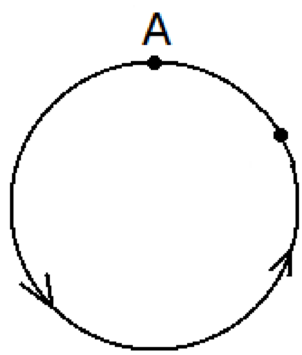
Graphically subtracting v's to find direction of a - 2

A ball moves at a constant speed in a counter-clockwise direction along a circular path. Determine the direction of the ball's acceleration when at position A.



Initial (earlier, before) Velocity: Draw an arrow representing the velocity of the ball before it arrives at point A. Label it v_i.

Final (later, after) Velocity: Draw an arrow representing the velocity of the ball after it passes point A. Label it v_f.

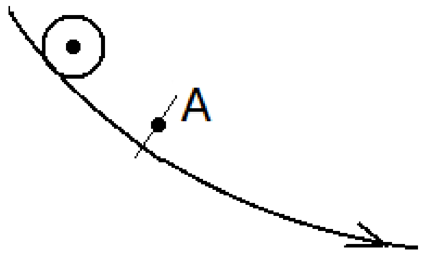


Velocity Change: Subtract the two velocity vectors to find $\overrightarrow{\Delta v} = \overrightarrow{v_f} - \overrightarrow{v_i}$. Put the tails together and draw the arrow that connects the head of v_i to the head of v_f. Label this new arrow Δv, "the change in velocity".

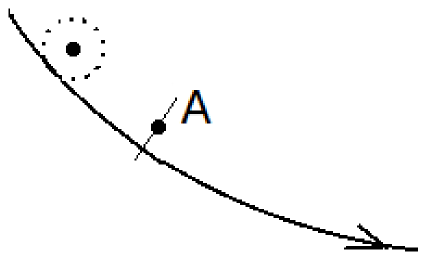
Acceleration: The acceleration equals the velocity change Δv divided by the time interval (between dots in the motion diagram). In this example, we only want the direction of a, which is <u>always</u> the same as the direction of Δv.
Go back to the original picture and draw an acceleration arrow and label it.

Graphically subtracting v's to find direction of a - 3

A marble passes point point A as it rolls along a slide with increasing speed (see sketch). Determine the direction of the marble's acceleration when it is at position A.



Initial (earlier, before) Velocity: Draw an arrow representing the velocity of the marble before it arrives at point A. Label it v_i.

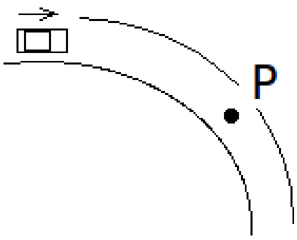
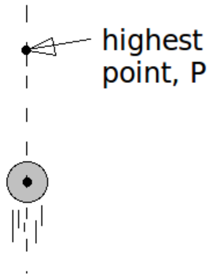
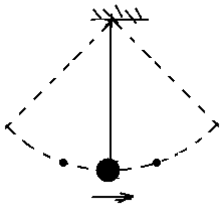
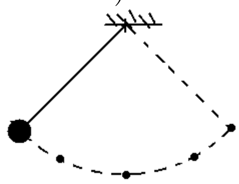
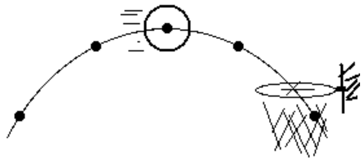


Final (later, after) Velocity: Draw an arrow representing the velocity of the marble after it passes point A. Label it v_f.

Velocity Change: Subtract the two velocity vectors to find $\overrightarrow{\Delta v} = \overrightarrow{v_f} - \overrightarrow{v_i}$. Put the tails together and draw the arrow that connects the head of v_i to the head of v_f. Label this new arrow Δv, "the change in velocity".

Acceleration: The acceleration equals the velocity change Δv divided by the time interval (between dots in the motion diagram). In this example, we only want the direction of a, which is always the same as the direction of Δv.
Go back to the original picture and draw an acceleration arrow and label it.

Graphically subtracting v's to find direction of a - 4

(a) A car travels with decreasing speed along a horizontal, curved road. Determine the direction of the acceleration when the car is at point P.  	(b) A large mass is attached to an oscillating spring. Determine the direction of the acceleration of the mass when it is at point P, moving downward with decreasing speed.  	(c) A ball is thrown vertically upward. Determine the direction of the acceleration of the ball when it is at its highest point. 
(d) A pendulum bob is swinging back and forth. Determine the direction of the acceleration when the bob is at its lowest point. 	(e) A pendulum bob is swinging back and forth. Determine the direction of the acceleration when it is at its highest point (at 45° as shown). 	(f) A basketball is thrown in an arc toward the basket. Determine the direction of the acceleration of the mass when it is at the highest point. 

1.3 Rulers and Clocks: Coordinate Systems

1.3.1 Signs of kinematic qualities - Straight-line motion

– Back to Straight-line Motion –

Kinematic quantities are things that describe the motion, such as time, position, velocity, and acceleration.

Up to now, there has been little reference to rulers or clocks.[1] There were no axes or coordinate systems. Instead, we used terms like "upward", "to the right", "earlier", or "at point A". One result is that our kinematic quantities were not "positive" or "negative". We deliberately didn't say things like "down is negative", because it wasn't.

To get numerical results, we must now introduce a clock and a ruler into our system. The ruler will define a "coordinate system". Of course, the clock will measure time and the ruler will measure position in space (where it is along the line).

The clock and ruler are *constructs* (invented by humans), and are not part of the physical system. (1) We do have a choice in how we place the ruler or when we start our clock. (2) Creating such constructs shouldn't change the physical system.

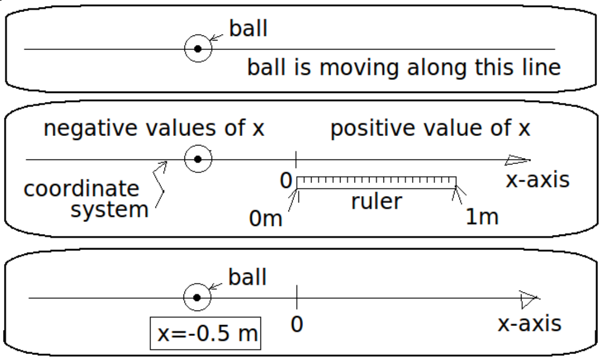


Figure 1.5: **top:** A ball is moving along a line, but its x coordinate value is not defined. **center:** Introduce a coordinate system by laying down a ruler. **bottom:** Identify the location of the ball. Here it is on the negative side of the x-axis.

Velocity and acceleration will fit nicely into this construction as well. But first time:

<u>**Time:**</u> We should be able to start the clock whenever we please. It is our choice. However, for convenience, it is usually, but not always, started at the beginning of the process. Any other event that happens when we start the clock will have $t = 0$. Any event that happens after that will have a positive time, $t > 0$. Earlier events will have negative time, $t < 0$.

[1] There was reference to measuring the distance between two points [how many meters did it travel], or the time interval between two events.

Position: See Figure [1.5]. We have two choices for the ruler. First, we can place the zero (origin). Second, we can align it in one of the two directions along the line defining the motion.

If the object is at our chosen origin, we say that $x = 0$.

The rest of the ruler will then lie along the positive x-axis. This side of the origin will correspond to positive, or increasing, values of x. If the object is on the positive x-axis, it will have a positive value of x, i.e., $x > 0$.

On the other side of the origin, the values of x will be negative. Fifty centimeters away from the origin, on the negative x-axis, it will be $x = -50\,\mathrm{cm}$.

So, the sign of x will depend on which side of the origin it is, and which way the axis points. We can now be in a position to say "up is positive", or "down is positive", depending on how we choose to position the ruler.

NOTE 1: The coordinate axis is labeled "x-axis", or the abbreviation "x".

NOTE 2: Only one arrowhead is drawn on the axis, on the positive x side.

NOTE 3: The origin must be clearly identified with a tick mark and a "0".

Velocity: For motion along a line, we have three things that are very often confused,

- velocity, $\vec{v}$ (a vector)
- speed, magnitude of the velocity, $v = |\vec{v}|$
- x-component of the velocity, v_x

The **velocity** is a vector, and strictly speaking, it is meaningless to say that it is positive or negative. It is represented by an arrow that points in a given direction. The arrow doesn't have a plus or minus sign.

The **speed** is the magnitude of the velocity, and it can never be negative. It can only be zero (if the object isn't moving), or positive. It is the answer to the question, "how many meters per second is the object moving". The object can never move a negative number of meters.

Both the velocity and speed are defined without reference to coordinate systems.

The **x-component of the velocity** is the projection of the velocity onto the x-axis. For this, there must be a coordinate system, with an x-axis.

The x-component of the velocity is defined to be positive if the velocity (vector) points in the same direction as the positive x-axis.

The x-component of the velocity is defined to be negative if the velocity (vector) points in the opposite direction as the positive x-axis.

Otherwise, the x-component of the velocity, v_x, is very similar to the speed v. It is a measure of how fast the object is moving, in meters per second. The difference is that it takes into account the object's direction of motion by way of a sign. This is one reason why it is confusing.

Another reason for the confusion is that many books and teachers refer to the sign of the velocity: "Is the velocity positive or negative?" when they really mean "Is the x-component of the velocity positive or negative."

Acceleration: Similar to velocity, there is some confusion. However, most confusion centers on the question, "Is $g = 9.8\,\mathrm{m/s^2}$, or is $g = -9.8\,\mathrm{m/s^2}$?" But, the confusion not only relates to gravity.

For our problem, we have three things:

- acceleration, $\vec{a}$ (a vector)
- magnitude of the acceleration, $a = |\vec{a}|$
- x-component of the acceleration, a_x

Note that we don't have a simple word analogous to "speed", for the magnitude of the acceleration.

Almost the same words apply for acceleration as they did for velocity.

The **acceleration** is a vector, represented by an arrow, and without a sign.

The **magnitude of the acceleration** is never negative.

Both of those are defined without a coordinate system.

The **x-component of the acceleration** is the projection of the acceleration onto the x-axis.

The x-component of the acceleration is defined to be positive if the acceleration (vector) points in the same direction as the positive x-axis.

The x-component of the acceleration is defined to be negative if the acceleration (vector) points in the opposite direction as the positive x-axis.

To resolve the issue with the sign of g, we will use g as the *magnitude* of the acceleration due to gravity (which is a downward pointing vector). g is never negative.

If a ball is thrown in any direction, while it is moving through the air[2], it will have a downward acceleration $\vec{a}$ whose magnitude is $a = g = 9.8\,\mathrm{m/s^2}$.

If we introduce a coordinate system, pointing upward, then $a_x = -g = -9.8\,\mathrm{m/s^2}$. However, if it points downward, then $a_x = +g = +9.8\,\mathrm{m/s^2}$.

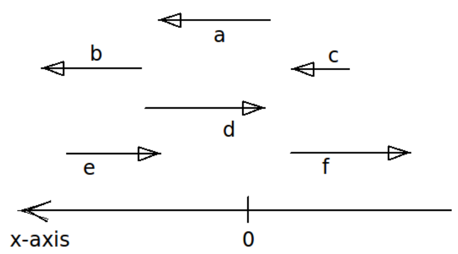


Figure 1.6: You are given six vectors (acceleration or velocity) and a coordinate axis. Decide if the sign of the x-component of each of these vectors is positive or negative or zero.

$a_x =$	$-$	0	$+$
$b_x =$	$-$	0	$+$
$c_x =$	$-$	0	$+$
$d_x =$	$-$	0	$+$
$e_x =$	$-$	0	$+$
$f_x =$	$-$	0	$+$

[2]... near the surface of the earth, and uninfluenced by anything except gravity of the Earth...

Table 1.3: **Signs of Kinematic Quantities.**

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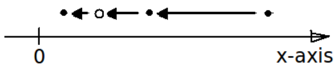
(a) A partial motion diagram that represents the motion of a car is shown below. Determine the sign (−0+) of the position, velocity, and acceleration of the car at the position of the open circle.



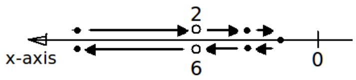
Position: − 0 +
Explain:

Velocity: − 0 +
Explain:

Acceleration: − 0 +
Explain:

</td>
<td>

(b) A partial motion diagram that represents the motion of a ball is shown below. Determine the sign (−0+) of the position, velocity, and acceleration of the ball at the positions of each open circle. Determine the signs of the displacement and change in velocity as the ball moves from position two to position six.



Position 2: − 0 +

Position 6: − 0 +

Velocity 2: − 0 +

Velocity 6: − 0 +

Acceleration 2: − 0 +

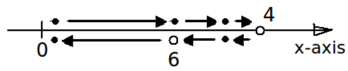
Acceleration 6: − 0 +

Displacement from two to six (2→6): − 0 +

Velocity change from two to six (2→6): − 0 +

</td>
<td>

(c) A partial motion diagram that represents the motion of a ball is shown below. Determine the sign (−0+) of the position, velocity, and acceleration of the ball at the positions of each open circle. Determine the signs of the displacement and change in velocity as the ball moves from position four to position six.



Position 4: − 0 +

Position 6: − 0 +

Velocity 4: − 0 +

Velocity 6: − 0 +

Acceleration 4: − 0 +

Acceleration 6: − 0 +

Displacement from four to six (4→6): − 0 +

Velocity change from four to six (4→6): − 0 +

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</table>

1.4 Constructing the Pictorial Representation

Most students that have trouble with physics say that they understand the material, but do not know how to get started on a problem. Constructing a pictorial representation of the problem is the best way to start such a problem.

To construct this representation, picture yourself in the situation described in the problem. Try to recreate the sequence of events that occurs, and make a sketch that indicates the major steps in the process. In kinematics, the problem is usually divided into parts separated by instants at which the acceleration changes.

See the figure for an example with only one interval: Just after ball leaves hand to just before ball hits ground.

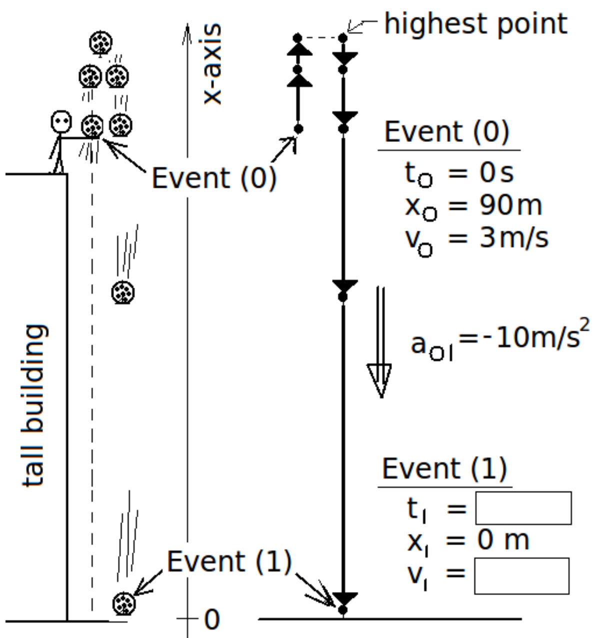


Figure 1.7: A person on top of a tall building throws a ball straight up as shown. The ball leaves his hand 90 m above the ground with a speed of 3.0 m/s^2.

Each part of the problem involves a description of the motion while the acceleration is approximately constant. The next part of the problem starts when the acceleration has changed because forces acting on the object have changed.

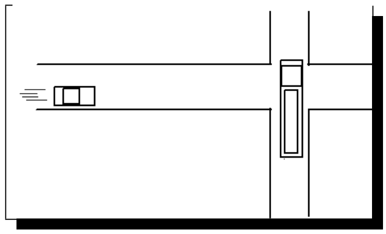
Some problems involve two objects that move either together or move independently. Just as characteristics of two people are describe differently, the characteristics of these objects must be described separately. Different symbols are used to indicate the position, velocity, acceleration, mass and other properties of each object.

The pictorial representation clearly identifies, in symbolic form, the times, position, and velocities of each object at events where changes in force or acceleration occur. The acceleration during each time interval (between successive events), as well as other known quantities, should also be represented symbolically in the pictorial representation. A separate set of kinematic quantities is needed for each object involved in the problem.

Pictorial Representation Checklist

- Sketch the system. Draw the objects at several important locations, and a few in-between. Qualitatively indicate any motion. Indicate where on each object you will measure location.
- Identify in your sketch, the important events. These are events between which the acceleration is constant. There may be other types of events, for example, when a ball reaches it's highest point, or where the cars pass each other.
- Draw qualitative motion diagrams for your system. Include a series of dots, velocity arrows connecting these dots, and appropriate acceleration arrows. Label the arrows.
- Choose a coordinate axis if one is not given. It should have a well-defined origin (where $x = 0$), and a positive direction. Draw the coordinate axis.
- Decide on when the clocks will start, if not specified in the problem. Perhaps you get to choose $t = 0$ for when the first event (labeled "Event (0)") starts.
- For each important event, name and tabulate the time, position, and velocity. Between the important events, name and tabulate the acceleration. Use the values (numbers and units) from the problem statement and from choices you made, such as placing rulers and starting clocks.
- Indicate any relationships between the kinematic quantities, if any. For example, if a car (using time t) and a truck (using time T) are passing each other, then $t_1 = T_1$ might be the time when they pass each other.
- Indicate any unknown kinematic quantities with an empty box. Relate each empty box to a [possible] question in the original problem statement. If you know the (non-zero) sign ($+/-$) of a kinematic quantity, identify that outside of your empty box.

Pictorial Representation – 1

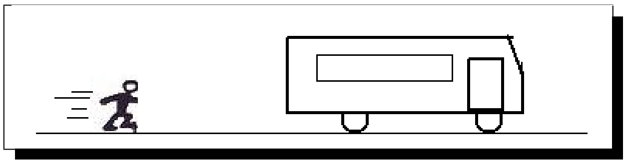


A driver in a car, traveling at a constant 16 m/s (35 MPH) on a through street suddenly sees a truck in front that has entered from a side street and that now blocks the car's path. What is the shortest stopping distance for the car assuming the reaction time of the driver is 0.75 seconds. The car's maximum acceleration is $6.0\,\mathrm{m/s^2}$.

(a) Construct a pictorial model for the problem, but do not solve it.

(b) For each kinematic unknown, identify the question, or write a question.

Pictorial Representation – 2



A woman runs at a constant speed of 5.0 m/s toward a waiting bus. When she is 8.0 m behind the open door of the bus, it starts to move forward at increasing speed, with an acceleration of 1.0 m/s^2. What time interval is needed for the woman to reach the open door?

(a) Construct a pictorial model for the problem, but do not solve it.

(b) For each kinematic unknown, identify the question, or write a question.

Pictorial Representation – 3

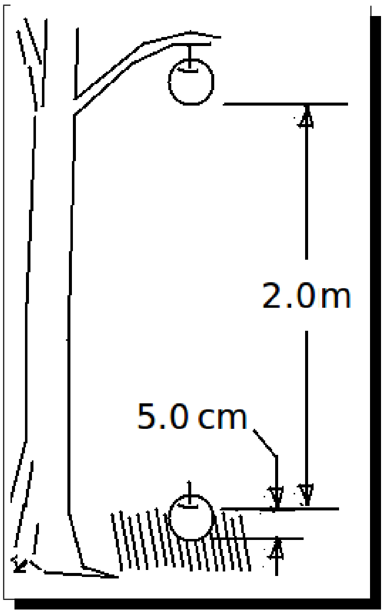
A car, initially at rest, accelerates toward the east at $3.0\,\mathrm{m/s^2}$. At the same time that the car starts, a truck 500 m to the east of the car and moving at 32 m/s toward the west starts to move slower, losing speed at a rate of $2.0\,m/s^2$. At what position, and at what time, will the car and truck pass each other?

(a) Construct a pictorial model for the problem, but do not solve it.

(b) For each kinematic unknown, identify the question, or write a question.

Pictorial Representation – 4

An apple falls from a tree from a distance of 2.0 m above the top of the grass below. While falling, it has a downward acceleration of 9.8 m/s^2. As the apple sinks into the grass, its speed decreases until it stops 0.050 m (5.0 cm) down into the grass, as shown. Assume the apple has a constant acceleration while it slows to a halt in the grass.



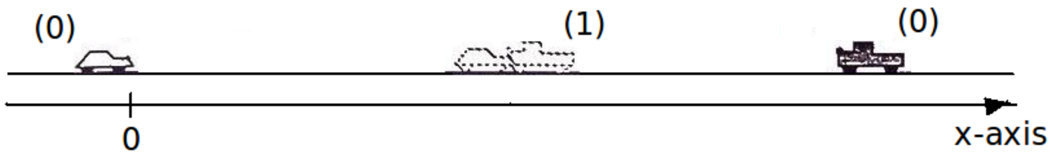
(a) Construct a pictorial model for the problem, but do not solve it.

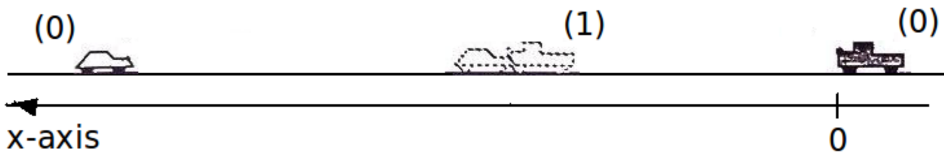
(b) For each kinematic unknown, write a question.

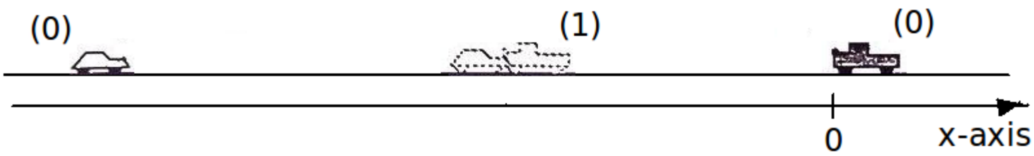
Pictorial Representation – 5

A car, initially at rest, accelerates toward the east at 3.0 m/s^2. At the same time that the car starts, a truck, 500 m east of the car, and moving at 32 m/s toward the west, starts to move slower, losing speed at a rate of 2.0 m/s^2. At what position will the car and truck pass each other? (Make the two motion diagrams.)

For this problem, several different coordinate axes are provided. For each, tabulate the known and unknown information. Suggestion: Use t, x, v, and a for the car; and T, X, V, and A for the truck.



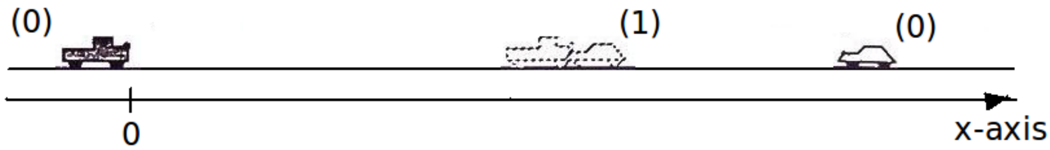


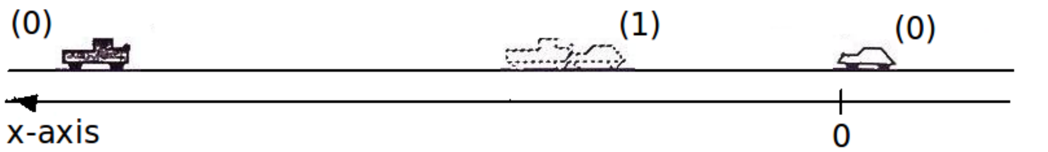


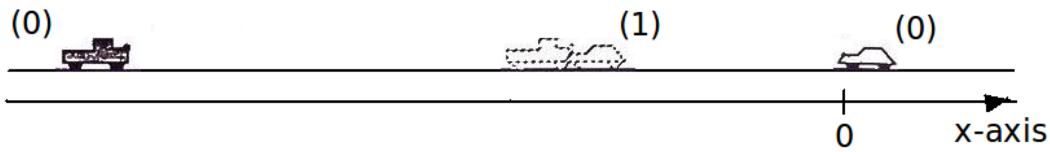
Pictorial Representation – 6

A car, initially at rest, accelerates toward the west at $2.0\,\mathrm{m/s^2}$. At the same time that the car starts, a truck, 1000 m west of the car, and moving at 16 m/s toward the east, starts to move slower, losing speed at a rate of $1.0\,\mathrm{m/s^2}$. At what position will the car and truck pass each other? (Make the two motion diagrams.)

For this problem, several different coordinate axes are provided. For each, tabulate the known and unknown information. Suggestion: Use t, x, v, and a for the car; and T, X, V, and A for the truck.



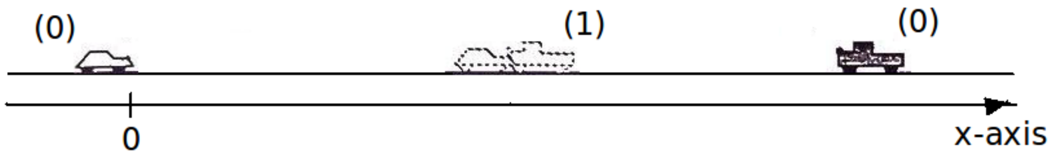


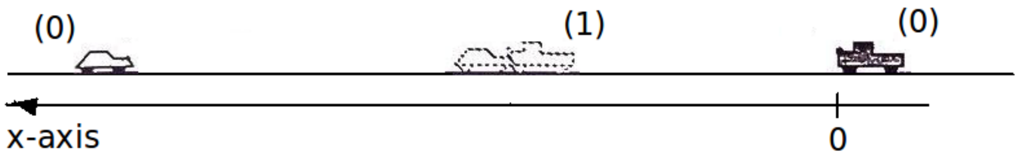


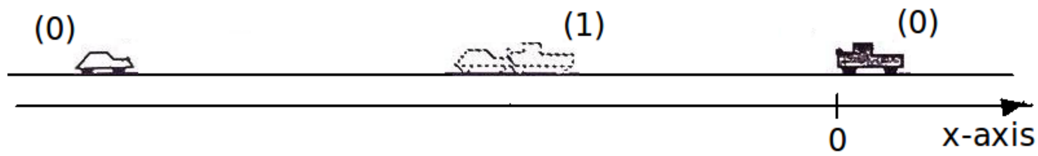
Pictorial Representation – 7

A car, initially moving east at 12 m/s, accelerates toward the east at 1.0 m/s^2. At the same initial time, a truck 1000 m east of the car, and moving at 18 m/s toward the west, starts to move slower, losing speed at a rate of 2.0 m/s^2. At what position will the car and truck pass each other? (Make the two motion diagrams.)

For this problem, several different coordinate axes are provided. For each, tabulate the known and unknown information. Suggestion: Use t, x, v; and a for the car, and T, X, V, and A for the truck.

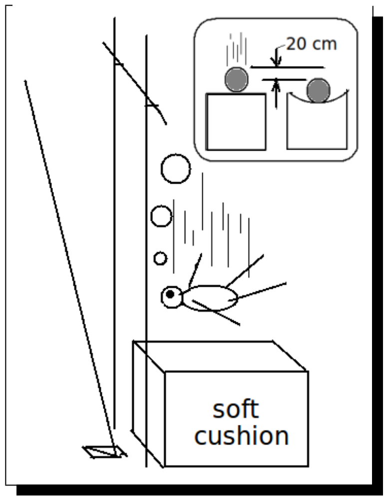






Multiple Representation Problem Solving – 1 (Pole Vaulter)

A pole vaulter, just before touching the cushion on which he lands after a jump, is falling downward at a speed of 10 m/s. The vaulter sinks about 0.20 m into the cushion before stopping. Estimate the average acceleration of the vaulter while stopping.



Pictorial Representation:
Construct a pictorial representation of the situation described in the problem. Include a coordinate axis, a sketch of the situation just before the vaulter touches the cushion and a sketch of the situation at the instant the vaulter stops; symbols that represent the of kinematic quantities, and their values, or empty boxes if unknown.

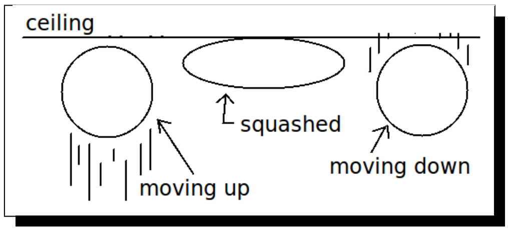
Physical Representation:
In your diagram above, construct a motion diagram for the vaulter while stopping as he sinks into the cushion. Check the signs of the known quantities in the sketches above by comparing them to the arrows in the motion diagrams.

Math Representation:
Chose one or more of the kinematic equations that relate the variables involved in the problem.

Solution:
Rearrange the equations so that the unknowns are alone on one side, and the known quantities are on the other side. Substitute the known information to determine the answer to the problem.

Evaluation: Check your answer for consistency. Do the velocity and acceleration signs, your chosen axis, and the motion diagrams all agree?
Is the unit of the answer correct? • Is the magnitude reasonable? • Did you answer the questions?
Go back and fill in the empty boxes with the values you figured out.

Multiple Representation Problem Solving – 2 (Ball Bounce)



Just before a ball hits the ceiling of a room, it is moving up with a speed of 6.0 m/s. After a time interval of 0.10 s, the ball leaves the surface of the ceiling and is moving down at a speed of 5.0 m/s. Determine the average acceleration of the ball during this time interval while in contact with the ceiling.

Pictorial Representation:
Construct a pictorial representation of the situation described in the problem. Include: a coordinate axis (for consistency, have the axis pointing up), a sketch of the situation just before the ball hits the ceiling and a sketch of the situation just after the ball leaves the ceiling, symbols that represent the known values of kinematic quantities in these sketches (be careful of signs), and a symbol representing the unknown that you wish to determine.

Physical Representation:
In your diagram above, construct a motion diagram for the ball. Represent the initial and final velocities with arrows. Graphically subtract these velocities to determine the direction of the acceleration. Check the signs of the known kinematic quantities in the sketches above against the directions of the arrows.

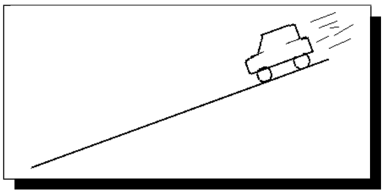
Math Representation:
Chose one or more of the kinematic equations that relate the variables involved in the problem.

Solution:
Rearrange the equations so that the unknowns are alone on one side, and the known quantities are on the other side. Substitute the known information to determine the answer to the problem.

Evaluation: Check your answer for consistency. Do the velocity and acceleration signs, your chosen axis, and the motion diagrams all agree?
Is the unit of the answer correct? • Is the magnitude reasonable? • Did you answer the questions?
Go back and fill in the empty boxes with the values you figured out.

Multiple Representation Problem Solving – 3 (Car on Hill)

The velocity of a car decreases as it travels down a hill that points down and to the left. Initially, it travels at a speed of 9.0 m/s. A little later, it is 20 m down the hill moving at 4.0 m/s.



Identify the unknowns and write questions for each of them. Here is one: What is the acceleration (both magnitude and direction) of the car?

Pictorial Representation:
Construct a pictorial representation of the situation described in the problem. Include: a coordinate axis (use one that is parallel to the surface of the hill), a sketch that shows the car at the initial and final situations described in the problem, symbols that represent the known values of kinematic quantities in these sketches (be careful of signs), and a symbol representing the unknown that you wish to determine.

Physical Representation:
Construct a motion diagram for the car during this time interval. Represent the initial and final velocities with arrows. Use the directions of the arrows in the motion diagram to Check the signs of the known kinematic quantities in your pictorial representation.

Math Representation:
Chose one or more of the kinematic equations that relate the variables involved in the problem.

Solution:
Rearrange the equations so that the unknowns are alone on one side, and the known quantities are on the other side. Substitute the known information to determine the answer to the problem.

Evaluation: Check your answer for consistency. Do the velocity and acceleration signs, your chosen axis, and the motion diagrams all agree?
Is the unit of the answer correct? • Is the magnitude reasonable? • Did you answer the questions?
Go back and fill in the empty boxes with the values you figured out.

Multiple Representation Problem Solving – 4 (Rocket Sled)

A rocket sled used to test automobile restraining devices (seat belts and air bags) accelerates from rest at $8.0\,\mathrm{m/s^2}$ for a distance of $16\,\mathrm{m}$. In order to simulate an emergency stop, the rocket sled is then decelerated at $64\,\mathrm{m/s^2}$.

Figure 1.8: Dr. Stapp, an American Air Force flight surgeon who also advocated mandatory seat-belts in cars, which became law in 1966.

Identify the unknowns and write questions for each of them. Here is one: What is the time interval needed to decelerate the sled to a stop?

Pictorial Representation:
Construct a pictorial representation of the situation described in the problem. Include: a coordinate axis, a sketch that shows the sled at the initial and final situations described in the problem, symbols that represent the known values of kinematic quantities in these sketches (be careful of signs), and symbols representing the unknowns that you wish to determine.

Physical Representation:
Construct a separate motion diagram for each part of the problem. Use the directions of the arrows in the motion diagrams to check the signs of the quantities in the pictorial representation.

Math Representation:
Chose one or more of the kinematic equations that relate the variables involved in the problem. You will have two sets of equations, one for each interval.

Solution:
Rearrange the equations so that the unknowns are alone on one side, and the known quantities are on the other side. Substitute the known information to determine the answer to the problem.

Evaluation: Check your answer for consistency. Do the velocity and acceleration signs, your chosen axis, and the motion diagrams all agree?
Is the unit of the answer correct? • Is the magnitude reasonable? • Did you answer the questions?
Go back and fill in the empty boxes with the values you figured out.

Multiple Representation Problem Solving – 5 (Car vs Truck)

The driver of a car traveling at 16 m/s sees a truck 20 m ahead coming straight toward her with a constant speed of 12 m/s. Without any delay (assume human reaction time is zero), she immediately begins to decelerate at 8.0 m/s^2. The truck driver is talking on his cell phone and is unaware of the impending collision, so he doesn't slow down.

Identify the unknowns and write questions for each of them. Here is one: If they don't swerve, when will they collide?

Pictorial Representation: Construct a pictorial representation of the initial situation (when the truck is first seen) and the final situation (when at same position). Include: a sketch that shows the car and truck at the initial and final situations, symbols that represent the known values of kinematic quantities in these sketches (be careful of signs), and symbols representing the unknowns that you wish to determine.

Physical Representation: Construct a separate motion diagram for the car and the truck. Use the directions of the arrows in the motion diagrams to check the signs of the quantities in the pictorial representation.

Math Representation: Write an equation that could be used to determine the position of the car at any time after the initial time.
Write and equation that could be used to determine the position of the truck at any time after the initial time.

Solution: The car and truck, if continuing to move straight toward each other, will meet (have the same x value). Use the equations above to determine when that happens.

Evaluation: Check your answer for consistency. Do the velocity and acceleration signs, your chosen axis, and the motion diagrams all agree?
Is the unit of the answer correct? • Is the magnitude reasonable? • Did you answer the questions?
Go back and fill in the empty boxes with the values you figured out.

Multiple Representation Problem Solving – 6 (Thrown Ball)

A ball is thrown straight down from a ledge to a person 20 m below the ball's starting position. The ball's initial downward speed is 5.0 m/s. Assume the acceleration of gravity is 10 m/s^2, and ignore effects due to the air.

Identify the unknowns and write questions for each of them. Here is one: What time interval must the person below have to react in preparation for the ball's arrival?

Pictorial Representation:
Construct a pictorial representation of the situation described in the problem. Include: a coordinate axis, a sketch that shows the ball at the initial and final situations, symbols that represent the known values of kinematic quantities in these sketches (be careful of signs), and symbols representing the unknowns that you wish to determine.

Physical Representation:
Construct a motion diagram for the ball. Use the directions of the arrows in the motion diagrams to check the signs of the quantities in the pictorial representation.

Math Representation:
Select an equation that could be used to determine the unknowns.

Solution:
Solve the equation for the unknowns. Recall that the quadratic $at^2 + bt + c = 0$ has the two solutions

$$t_{\pm} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

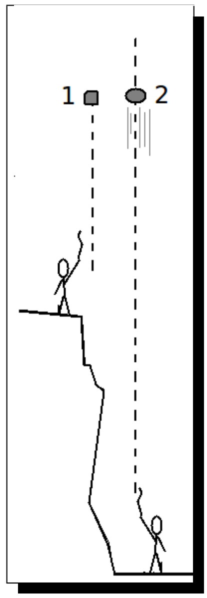
Interpret the "negative" time.

Evaluation: Check your answer for consistency. Do the velocity and acceleration signs, your chosen axis, and the motion diagrams all agree?
Is the unit of the answer correct? • Is the magnitude reasonable? • Did you answer the questions?
Go back and fill in the empty boxes with the values you figured out.

Multiple Representation Problem Solving – 7 (Two Stones)

A person standing on a cliff uses a sling to shoot a stone vertically upward with an initial speed of 30 m/s. Simultaneously, a person on a ledge 15 m below shoots a second stone vertically upward. The situation is such that the second stone is shot with such a speed that it hits the first stone at the apex of the first stone's flight. Assume the acceleration of gravity is $10\,\mathrm{m/s^2}$, and ignore effects due to the air.

Identify the unknowns and write questions for each of them. Here is one: How fast must the second stone be shot?



Pictorial Representation:
Construct a pictorial representation of the initial and final situations for each stone. The final situation is when the stones are side-by-side at the top (of the first stone's flight). Include:a coordinate system, a sketch that shows both of the stones at the initial and final situations, symbols that represent the known values of kinematic quantities in these sketches (be careful of signs), and symbols representing the unknowns that you wish to determine.

Physical Representation:
Construct a motion diagram for each stone. Use the directions of the arrows in the motion diagrams to check the signs of the quantities in the pictorial representation.

Math Representation:
Write equations that could be used to determine the position and velocity of the first stone at any time after the initial time.
Write equations that could be used to determine the position and velocity of the second stone at any time after the initial time.

Solution:
Solve the equations for the unknowns. You may need to algebraically combine several equations.

Evaluation: Check your answer for consistency. Do the velocity and acceleration signs, your chosen axis, and the motion diagrams all agree?
Is the unit of the answer correct? • Is the magnitude reasonable? • Did you answer the questions?
Go back and fill in the empty boxes with the values you figured out.

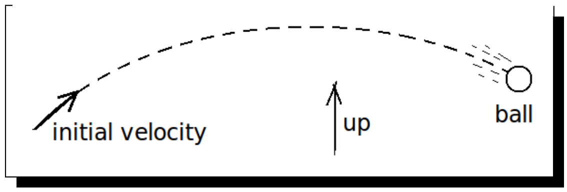
Changing Kinematic Representations – 1

(a) An object moves along a horizontal surface. The application of two kinematic equations to the motion is shown below. Construct a motion diagram representing the motion and then invent some real process that might be represented by the equations and by the motion diagram. $$x = 0 + (12\,\mathrm{m/s})(2.0\,\mathrm{s}) + \frac{1}{2}\,a\,(2.0\,\mathrm{s})^2$$ $$0 = (12\,\mathrm{m/s}) + a\,(2.0\,\mathrm{s})$$	(b) An object moves vertically. The application of two kinematic equations to that motion is shown below. Construct a motion diagram representing the motion and then invent some real process that might be represented by the equations and by the motion diagrams. $$v_1^2 - 0 = 2\,(-10\,\mathrm{m/s^2})\ [(0) - (5.0\,\mathrm{m})]$$ $$0 - v_1^2 = 2a_{12}\ [(-0.020\,\mathrm{m}) - (0)]$$
Solve the equations for the unknowns.	Solve the equations for the unknowns.
Draw a motion diagram that is consistent with the kinematic quantities and with the motion described by the equations. The object is moving horizontally.	Draw a motion diagram that is consistent with the kinematic quantities and with the motion described by the equations. The object is moving vertically. (You will need two motion diagrams here.)
Construct a pictorial representation of some process that is consistent with the motion diagram and the equations above. (There are many possibilities.)	Construct a pictorial representation of some process that is consistent with the motion diagram and the equations above. (There are many possibilities.)

Changing Kinematic Representations – 2

(a) An object moves along a horizontal surface. The application of three kinematic equations to the motion is shown below. Construct a motion diagram representing the motion and then invent some real process that might be represented by the equations and by the motion diagram. $$9.0\,\mathrm{m} - 0 = 0 + \frac{1}{2}\,(2.0\,\mathrm{m/s^2})\,t_1^2$$ $$v_1 = (2.0\,\mathrm{m/s^2})\,t_1$$ $$36\,\mathrm{m} - 9.0\,\mathrm{m} = v_1\,(t_2 - t_1)$$	(b) An object moves on an incline. The application of two kinematic equations to that motion is shown below. Construct a motion diagram representing the motion and then invent some real process that might be represented by the equations and by the motion diagrams. $$(12\,\mathrm{m/s})^2 - (24\,\mathrm{m/s})^2 = 2\,(-8.0\,\mathrm{m/s^2})\,(x - 0)$$
Solve the equations for the unknowns.	Solve the equation for the unknown.
Draw a motion diagram that is consistent with the kinematic quantities and with the motion described by the equations. The object is moving horizontally. [You need 2 diagrams.]	Draw a motion diagram that is consistent with the kinematic quantities and with the motion described by the equations. The object is moves along an incline plane (斜面).
Construct a pictorial representation of some process that is consistent with the motion diagram and the equations above. (There are many possibilities.)	Construct a pictorial representation of some process that is consistent with the motion diagram and the equations above. (There are many possibilities.)

1.5 Kinematics of Projectile Motion



EXAMPLE: Construct a motion diagram for a projectile that initially moves in a diagonal direction (斜线) above the horizontal – like a baseball (棒球) after leaving a bat (棒球棒) or a football (美国式足球) after leaving the foot of the punter (踢悬空求的队员). Ignore effects due to the air.

To construct the motion diagram, combine two step-by-step motion diagrams, one for the horizontal motion and one for the vertical motion. As we construct the motion diagrams, assume the time between dots is the same for both motions.

Construct a <u>horizontal motion diagram</u> for an object $\mathcal{X}$ that moves at a constant speed in the horizontal direction from left to right. Its ($\mathcal{X}$'s) horizontal speed is the horizontal component of the ball's initial velocity. Include 7 dots and 6 velocity arrows. Label the velocity v_x. Don't forget "$(a_x = 0)$".

Construct a <u>vertical motion diagram</u> for an object $\mathcal{Y}$ with an initial upward velocity. Its ($\mathcal{Y}$'s) initial vertical speed is the vertical component of the ball's initial velocity. This object experiences a constant downward acceleration. Draw the up dots and arrows slightly to the left of the down dots and arrows, and put one dot at the top center. Make it symmetrical with three up velocity arrows, and three down velocity arrows. Label the velocity v_y. Don't forget the a_y arrow.

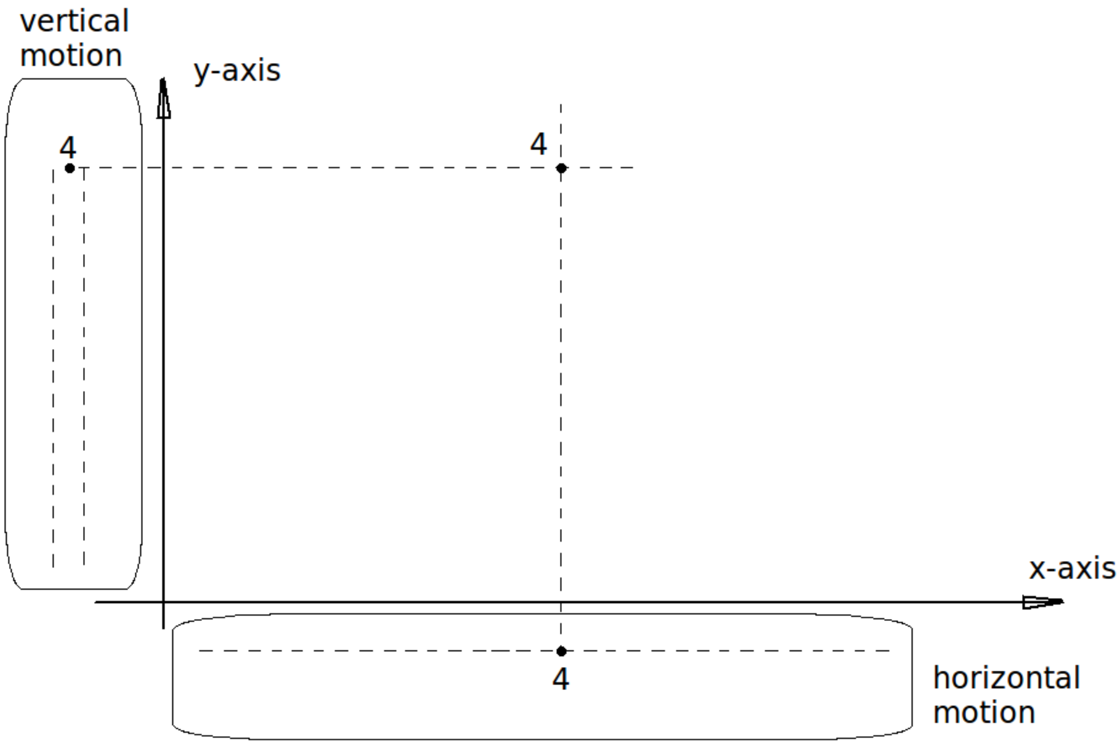
NOTE: The 2-D problem of <u>one</u> projectile is similar to the 1-D (straight-line) motion of <u>two</u> vehicles, one moving at a constant speed, and the other moving with a constant acceleration. The motion diagrams above should be very familiar.

In the following, as we combine them, think of the two motion diagrams above as representing the motions of shadows (cast a shadow:投下影子):

$\mathcal{X}$ is the ball's shadow on the floor cast from a light source far above the ball, and

$\mathcal{Y}$ is the ball's shadow on the wall cast from a light source far to the right of the ball.

Combine the two motion diagrams into a single two-dimensional (2-D) diagram. Number the dots in all three diagrams from 1 to 7. To make the correspondence, the middle dot (4) of $\mathcal{X}$ will be when the ball is at the highest point, dot (4) of $\mathcal{Y}$. Place the horizontal motion diagram along the x-axis, and the vertical motion diagram along the y-axis. Find out where the ball's "dots" would be in the xy-plane. The ball's velocity arrows are made by connecting the 7 dots in the plane. Don't forget the a arrow: The acceleration of the ball is everywhere downward (combine a_x and a_y : $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$)



A look ahead: According to Newton's laws, the direction of the ball's acceleration will be in the direction of the total force acting on the ball. Since the acceleration points straight down, which way will the total force point? ________________ What is that force called? ____________ What physical thing causes that force? _________________________

If we ignore effects due to the air, a projectile moves at a constant speed in the horizontal direction, while simultaneously (at the same time, 同时的) moving in the vertical direction with a downward pointing acceleration.

People before Newton thought (and many students today think) that a force must push the projectile in the direction of motion (to keep it moving through the air). HOWEVER, there is no other object that touches the projectile to push in that direction.

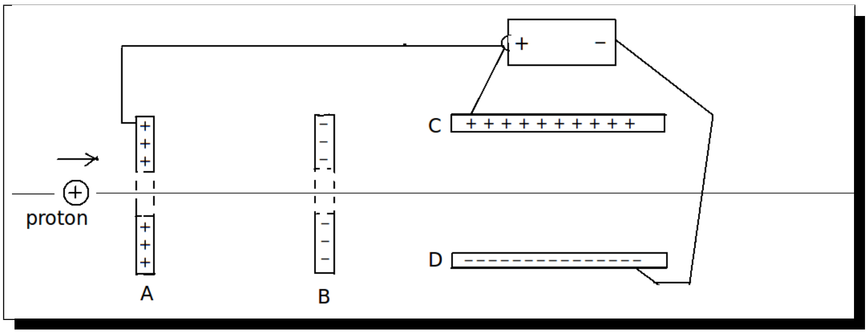
Projectile Motion – 1

Use the procedures described above to construct motion diagrams for these special situations. Assume the initial speed is $\boxed{v = 50\,\text{m/s}}$. Ignore effects due to the air. Calculate the components of the initial velocity, v_{x0} and v_{y0} as indicated. Assume x is to the right and y is upward.

(a) a projectile that initially moves in the horizontal direction to the right v_{x0}=__________ v_{y0}=__________	(b) a projectile that initially moves in a direction to the right but 37° below the horizontal v_{x0}=__________ v_{y0}=__________
(c) a projectile that initially moves in a direction to the right but 37° above the horizontal v_{x0}=__________ v_{y0}=__________	(d) a projectile that initially moves in a direction to the right but 57° above the horizontal v_{x0}=__________ v_{y0}=__________
(e) a projectile that initially moves in a direction to the left but 37° below the horizontal v_{x0}=__________ v_{y0}=__________	(f) a projectile that initially moves in a direction to the left but 57° below the horizontal v_{x0}=__________ v_{y0}=__________

In all of the above situations, what is the direction of the acceleration? __________

Proton as a Projectile – 2



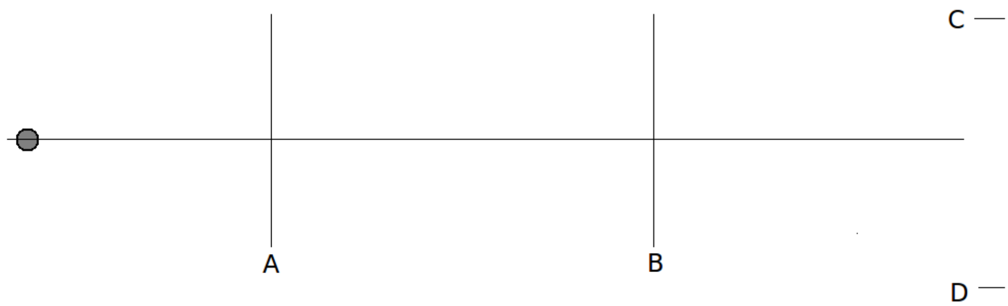
A proton (质子), with a small positive electric charge, initially moves toward the right, as shown. It moves into an area containing an arrangement of (electrically charged) plates connected to a battery.

Region A-B: It first passes into a region where it speeds up between charged plates A and B. Here the acceleration is constant and toward the right.

Region C-D: Then it passes into a region between plates C and D. Here the acceleration is constant and straight down.

In the regions outside of the plates, it moves with a constant velocity.

(a) Construct a motion diagram for the proton from its initial position to just before region C-D.



(b) Construct a motion diagram for the proton from the time it enters region C-D until it exits to the right.

In the top figure, lightly sketch the general path of the proton as it passes through the system.

1.5.1 Projectile Motion and Velocity Change

Remember This: When something is dropped, its downward acceleration is $10\,\text{m/s}^2$. This means that each second it falls, we must add $10\,\text{m/s}$ to its speed. Immediately after releasing, its speed is zero. After one second, it is $10\,\text{m/s}$. After two seconds, it is $20\,\text{m/s}$. After three seconds, it is $30\,\text{m/s}$. ... and so on. (Upward motion is symmetrical.)

(a) If a stone is dropped, and falls for $3.6\,\text{s}$ before it hits the ground, how fast is it moving just before it hits the ground? Final speed = ____________

(b) If we throw a stone straight up, and it takes two seconds for it to return, how fast was it thrown? Initial upward speed = ____________

The initial velocity of several projectiles are listed below. For each case, determine the initial x ($\rightarrow$) and y ($\uparrow$) velocity components, and the velocity components two seconds later. Assume $g = 10\,\text{m/s}^2$ and ignore effects due to the air. The arrow gives the general direction of the initial velocity. [See page 36.]

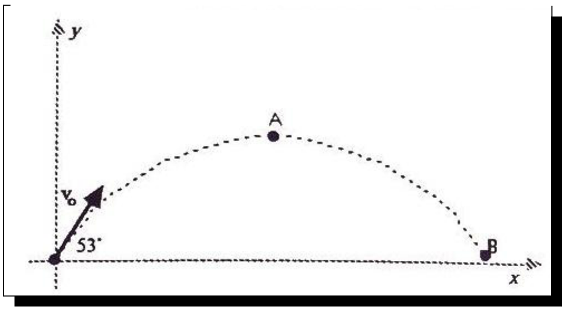
Initial Velocity	$v_x(0\,\text{s})$ (m/s)	$v_x(2\,\text{s})$ (m/s)	$v_y(0\,\text{s})$ (m/s)	$v_y(2\,\text{s})$ (m/s)
(a) $\rightarrow$, $50\,\text{m/s}$ horizontally				
(b) $\searrow$, $50\,\text{m/s}$, $37°$ below the horizontal				
(c) $\nearrow$, $50\,\text{m/s}$, $37°$ above horizontal				
(d) $\nearrow$, $50\,\text{m/s}$, $53°$ above horizontal				
(e) $\swarrow$, $50\,\text{m/s}$, $53°$ below horizontal				
(f) $\swarrow$, $50\,\text{m/s}$, $53°$ below horizontal				

Extra: When does the projectile in (c) reach its highest point? ____________________

What is the velocity of the projectile at that point? ____________________

Projectile Motion Question – 1

A ball is projected from the origin with initial velocity $\vec{v_0}$ as shown at the right. The initial speed of the ball is 50 m/s. Assume that $g = 10\,\text{m/s}^2$ and ignore effects due to the air.



Point A is at the highest point. Point B is where it returns to the ground level. [No calculator needed.]

(a) Determine the x component of the initial velocity. v_{0x}=__________

(b) Determine the y component of the initial velocity. v_{0y}=__________

(c) Determine the x component of the acceleration. a_x=__________

(d) Determine the y component of the acceleration. a_y=__________

(e) Complete the table below indicating the position and velocity at one-second time intervals beginning at $t = 0$ when the ball leaves the origin.

t (s)	$x(t)$ (m)	$v_x(t)$ (m/s)	$y(t)$ (m)	$v_y(t)$ (m/s)
0.0				
1.0				
2.0				
3.0				
4.0				
5.0				
6.0				
7.0				
8.0				

(f) Determine the ball's velocity at position A. $\vec{v}$(at A)=__________________

(g) Determine the location of position B. (x, y) at B __________________